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Simultaneous Signaling to the Capital
and Product Markets
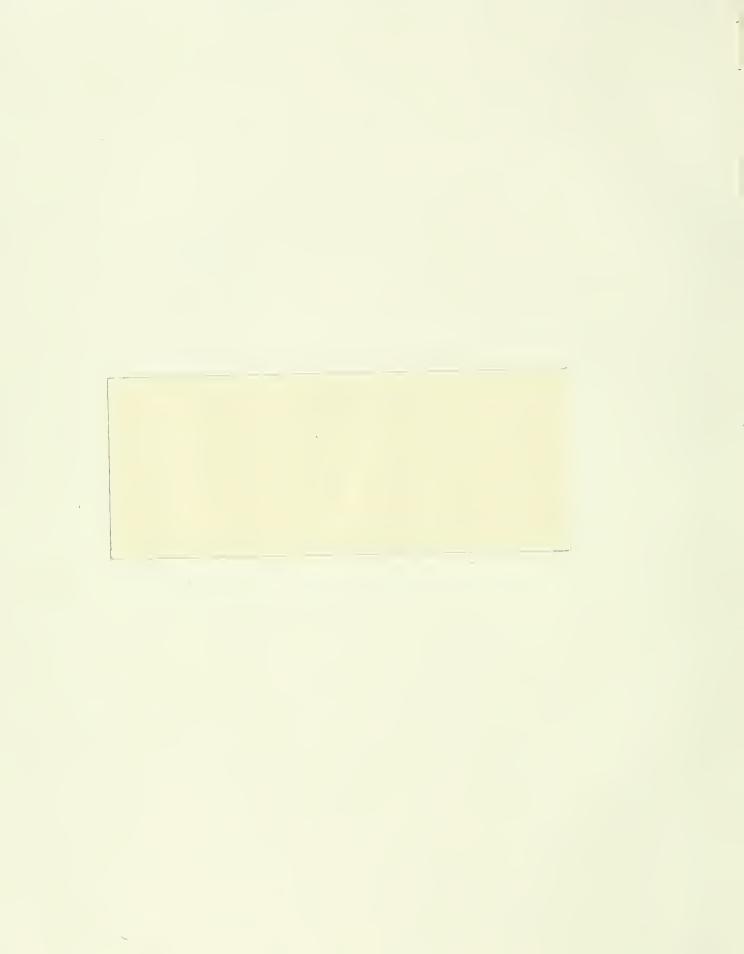by
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Abstract

This paper analyzes an informed firm's choice of financial structure using what we call a two-audience signaling model: the choice of a financing contract is observed not only by the capital market, but also by a second interested (but uninformed) party. This second party might be a competing firm or a labor union.

A key feature of the model is that the informed firm's gross profit is endogenous, because the second party's action depends on the transaction it observes between the informed firm and the capital market. The main result is that the reasonable capital-market equilibria (i.e., those that satisfy a refinement) maximize the ex-ante expectation of the informed firm's endogenous gross product-market profits. In this sense, the character of capital-market equilibrium is determined by the structure of the product market.

An immediate corollary is that (generically) either all the reasonable equilibria are separating or all the reasonable equilibria are pooling. Indeed, the latter is often the case. This is in distinct contrast to earlier work on the information content of financial structure and to more recent work on refinement in signaling games, both of which focus on separating equilibria.

1. Introduction

Models in which informed managers attempt to signal private information to the capital market have been used to explain a wide variety of corporate financial behaviors, including capital-structure decisions (Ross [1977], Myers and Majluf [1984]), dividend policy (Bhattacharya [1979], Miller and Rock [1985]) and management share ownership (Leland and Pyle [1977]). The appeal of these models is that they account for corporate financial behavior that is otherwise difficult to rationalize, and they are based on the reasonable assumption that a manager has private information about the firm's performance.

These models, like most in the finance literature, abstract from the other markets in which the firm operates. Yet, when a firm reveals information to the capital market, it often does so by a publicly observable action (such as a dividend) that reveals information to otherwise uninformed agents in other markets (such as product-market rivals). These agents then condition their behavior on this information, and this affects the profit (gross of financing costs) of the informed firm. When the informed firm's gross profit is endogenously determined in this way, product-market considerations affect the informed firm's decision to reveal information through its financial policy.

This payoff endogeneity is important; it is the crucial determinant of equilibrium in our model. It is also a second dimension (in addition to asymmetric information) on which the model departs from the Modigliani-Miller framework: financing costs and gross profits are endogenously determined by financial policy.

We capture these ideas by developing a model in which the informed firm signals to two uninformed audiences---the capital market and the

product market. Although such two-audience signaling models have received little attention in the literature, a rich collection of potential applications exists. We discuss some of these in the conclusion.

Our main result is that the reasonable capital-market equilibria (i.e., those that satisfy a refinement) maximize the ex-ante expectation of the informed firm's endogenous gross product-market profits. In this sense, the character of capital-market equilibrium is determined by the structure of the product market.

An alternative interpretation, however, is that the character of product-market equilibrium is dramatically affected by the existence of the capital market. In our model there are no opportunities to signal in the product market. Therefore, in the absence of the capital market, product-market equilibrium must be pooling. In the presence of the capital market, however, our main result shows that equilibrium can be either pooling or separating. Both of these interpretations suggest that it may be misleading to analyze the firm's financial side separately from its real side.

An immediate corollary of our main result is that (generically) either all the reasonable equilibria are separating or all the reasonable equilibria are pooling. Indeed, the latter is often the case. This is in distinct contrast to earlier work on the information content of financial structure and to more recent work on refinement in signaling games, both of which focus on separating equilibria. Our result shows that such full disclosure need not be (reasonable) equilibrium behavior.

To emphasize the effects that arise with the introduction of the product market, Section 2 presents an extremely simple one-audience

signaling model of financial policy when there is asymmetric information about the issuing firm's _exogenous_ gross payoffs. This model is in the spirit of Myers and Majluf [1984], who make the fundamental point that a firm's preference for debt over equity increases as the firm's profit increases, because the financing cost of an equity issue increases with the firm's profit. This motivates a negative inference about firm value when a firm issues equity. This inference is consistent with the empirical finding that share prices decrease after the announcement of an equity issue; see, for example, Asquith and Mullins [1986]. In the simple model in Section 2, we show that capital-market equilibrium can be either separating or pooling: in the separating equilibria, the low-profit firm issues more equity and the high-profit firm issues more debt; in the pooling equilibrium, both types of firm issue debt.

In Section 3, we present a general model of product-market competition under asymmetric information. This describes the relationship between the informed firm and the second audience. We then offer two examples that differ in one crucial respect. In each example there are two firms that play a Cournot game under asymmetric information: the issuing firm (firm A) knows the level of demand; its rival (firm B) does not know the level of demand, but observes firm A's transaction with the capital market. In the first example, the rival is an extant firm in the industry; in the second, the rival is a potential entrant. The crucial difference between the models is as follows. In the first, the ex-ante expected profit to firm A is greater when its capital-market transaction reveals nothing about demand (i.e., when capital-market equilibrium is pooling) than when its transaction reveals demand perfectly (i.e., when

capital-market equilibrium is separating). In the second example, the opposite is true.

Section 4 integrates the two relationships described in Sections 2 and 3. We explain the timing and information structure of our game, and define the refinement we impose. Section 5 proves our main result and states some additional characterization results. Our main result is that the form of capital-market equilibrium (i.e., whether this equilibrium is pooling or separating) maximizes the ex-ante expected product-market profit to firm A. If ex-ante expected product-market profit to firm A is higher when firm B is uninformed, capital-market equilibrium is pooling: firm A's choice of capital structure reveals no information. If ex-ante expected product-market profit to firm A is higher when firm B is informed, capital-market equilibrium is separating: firm A's choice of capital structure perfectly reveals its private information.

This result is in stark contrast to the results in models with exogenous payoffs: without product-market concerns, equilibrium can be pooling or separating (as shown in Section 2), whereas in our model the product market determines which form of equilibrium prevails. In addition, given a very mild regularity condition, we show that reasonable pooling equilibria must be balanced: if one type of the informed firm has higher gross profit when capital-market equilibrium is pooling than when it is separating, then that type must also have higher financing costs when capital-market equilibrium is pooling than when it is separating. An immediate corollary of this result is that there cannot exist a pooling equilibrium in which the firm issues only debt. This contrasts with the result in Section 2 which states that only debt is issued in a pooling equilibrium. The result also differs from Myers and Majluf [1984] and

Gertner [1986], who use models that abstract from product-market effects to show that only debt is issued in a pooling equilibrium.

We conclude this introduction by discussing some related work. Bhattacharya and Ritter [1983] also consider the informational links between the product and capital markets. They analyze a model in which a firm needs to raise capital for an R&D project. Information revealed to the capital market about the firm's technology is observed by the firm's uninformed rivals. Bhattacharya and Ritter show that there exists a separating equilibrium in which the better the firm's technology, the more of its technology it chooses to reveal.

There are several differences between the Bhattacharya-Ritter analysis and ours. First, we focus on an indirect mechanism (namely, capital structure) by which information is revealed, whereas they consider direct (and verifiable) information revelation. Second, they demonstrate that a particular separating equilibrium exists, while in our model pooling equilibria not only exist but may be more reasonable than any separating equilibrium. Finally, our model is closer in spirit to the asymmetric-information models used in the finance literature, and this enables us to identify the changes in equilibrium financial structure that occur when product-markets effects are included.

Glazer and Israel [1987] also consider the effect of financial signaling on product-market competition. They show that the choice among alternative compensation schemes by an informed manager of an incumbent monopolist can affect the entry decision of a potential competitor. In their model, however, the incumbent's shareholders contract with the manager before the manager becomes informed, so the subsequent game between the manager and the potential competitor is a standard one-

audience signaling problem.

Other papers that analyze the interplay between the product and capital markets include Allen [1986], Brander and Lewis [1986], and Maksimovic [1986]. These papers analyze the effect of bankruptcy in product-market games of complete information. Since a firm's capital structure affects its probability of bankruptcy, these models link the capital and product markets. There is no information transmission in these models, however, because they assume complete information. We ignore the possibility of bankruptcy so as to abstract from the effects these papers analyze and focus on the information content of financial structure.

2.   Signaling to Only the Capital Market

This section presents a signaling model of capital structure in the spirit of Myers and Majluf. Consider a firm (or entrepreneur) that needs to raise \$K to undertake a project. The gross profit from the project, π, is either π_H or π_L, where $\pi_H > \pi_L$; we refer to the firm's gross profit as its "type." The firm knows how profitable the project will be, but potential creditors do not.[1] Our analysis is independent of the number of potential creditors, so we assume that only one exists. Ex-ante, this creditor believes that the return will be π_H with probability $\phi \epsilon (0,1)$. Ex-post, realized profits are observable and verifiable by all parties, so financing contracts can be contingent on π.

We consider the following two-stage game between the firm and the creditor. In the first stage, the firm offers a financing contract involving debt or equity or both. In particular, a contract $(\alpha, D) \epsilon R^2$ means that the firm pays the creditor $\min \{D+\alpha(\pi-D), \pi\}$ when profit is π.

This payment function reflects our assumption that the entrepreneur is protected by limited liability: the firm can never be forced to pay creditors more than π. We interpret D as a debt payment and α as equity participation, although we simplify our analysis greatly by <u>not</u> requiring $D \geq 0$ or $\alpha \epsilon [0,1]$. This is equivalent to assuming that any linear contract between the firm and the creditor is feasible. In this two-type model, an arbitrary financing contract can be represented by a linear contract: all that is relevant are the two payments made by the two types of firm. Thus, in this simple setting, restricting attention to linear contracts is without loss of generality.

In the second stage of the game, given a financing contract offered in the first stage, the creditor chooses whether to accept or reject it. If the creditor accepts the contract, then the firm invests in the project and its profit is distributed according to the terms of the contract. If the creditor rejects the contract, the game ends.

The creditor is risk-neutral and can earn an expected rate of return r elsewhere. Therefore, the creditor makes non-negative expected profit from the contract (α,D) if $E[\min\{D+\alpha(\pi-D),\pi\}] \geq (1+r)K \equiv R$, where the expectation is taken with respect to the creditor's belief about the firm's profit, π_H or π_L. The firm's payoff if the contract (α,D) is accepted is $\max\{(1-\alpha)(\pi-D),0\}$; this payoff is deterministic because the firm knows the value of π. To keep things simple, we assume that gross profits are sufficient to avoid bankruptcy: $\pi_H > \pi_L > R$.

In this game, a (pure) strategy for the firm is a function from its type space, $\{\pi_H,\pi_L\}$, into the contract space, R^2. A (pure) strategy for the creditor is a function from the contract space, R^2, into an acceptance decision in $\{0,1\}$. We also need to define the belief held by

the creditor after observing the contract (α, D) offered by the firm: let $\mu(\alpha, D)$ be the creditor's assessment of the probability that $\pi = \pi_H$. A perfect Bayesian equilibrium in this game is a pair of strategies and a belief function satisfying two conditions:

(E1) (i) Given its type and given the creditor's strategy, the firm's strategy maximizes its expected payoff.

(ii) Given a contract offered by the firm, (α, D), and given the creditor's belief, $\mu(\alpha, D)$, the creditor's strategy maximizes its expected payoff.

(E2) When possible, the belief $\mu(\alpha, D)$ is derived from Bayes' rule and the firm's strategy.

In terms of the existing literature, this is a signaling game, as introduced by Spence [1973]. It is straightforward to show that the following pure-strategy pooling and separating equilibria exist. (We ignore mixed strategies in what follows). All of these equilibria correspond to intuitions developed by Myers and Majluf.

First, there is a unique pooling equilibrium in which both types of firm offer the debt contract $(0, R)$. This equilibrium is supported by the following beliefs off the equilibrium path: if the creditor observes the deviation (α', D'), its belief $\mu(\alpha', D')$ is zero if $\alpha' \geq 0$ and one if $\alpha' < 0$. These beliefs imply that, for any deviation (α', D'), either the creditor rejects the contract or neither type of firm has a strict incentive to offer it.

To see that no other pooling equilibria exist, suppose that (α, D) were a pooling contract, where $\alpha \neq 0$. Any pooling equilibrium must involve non-negative expected profit for the creditor:

$$\alpha\left[\phi\pi_H+(1-\phi)\pi_L-D\right] + D \geq R.$$

For $\alpha \neq 0$, at least one type of firm pays a financing cost strictly greater than R. This type can offer the deviation (0,R), which the creditor accepts for <u>any</u> belief μ. Clearly, the same deviation breaks a putative pooling equilibrium at (0,D), where D > R. Thus, (0,R) is the unique pooling perfect Bayesian equilibrium contract. This equilibrium formalizes part of the intuition behind the Myers-Majluf observation that issuing firms prefer debt to equity.

Second, there exists a continuum of (essentially equivalent) separating equilibria. In a separating equilibrium, the high-profit firm offers the contract (α_H, D_H) and the low-profit firm offers the (different) contract (α_L, D_L). Because each type of firm could offer the other's contract, the following incentive-compatibility constraints must hold:

$$(1-\alpha_H)(\pi_H-D_H) \geq (1-\alpha_L)(\pi_H-D_L) \quad \text{and}$$

$$(1-\alpha_L)(\pi_L-D_L) \geq (1-\alpha_H)(\pi_L-D_H).$$

(The argument below equation (4) in Section 5 shows that limited-liability constraints are not an issue here.) Adding these two inequalities yields $\alpha_L \geq \alpha_H$, and we can rule out $\alpha_L = \alpha_H$ because the incentive-compatibility constraints then imply $D_L = D_H$, which is pooling. Thus, $\alpha_L > \alpha_H$: in a separating equilibrium, the low-profit firm issues more equity than does the high-profit firm. This follows directly from the

Myers-Majluf observation that equity financing costs increase with gross profit.

A separating equilibrium also must guarantee the creditor non-negative expected profit. In fact, the creditor earns zero profit on each contract: as above, if a single type of firm pays a total financing cost greater than R, then it would offer $(0, R)$, which is always accepted. The creditor's zero-profit constraints are

$$\alpha_H \pi_H + (1-\alpha_H)D_H = R \quad \text{and}$$

$$\alpha_L \pi_L + (1-\alpha_L)D_L = R.$$

These equations reduce the incentive-compatibility constraints to a simple pair of inequalities: $\alpha_L \geq 0 \geq \alpha_H$. These inequalities and the creditor's zero-profit constraints completely characterize the continuum of separating equilibria. These equilibria are supported by the out-of-equilibrium beliefs described for the pooling equilibrium: $\mu(\alpha', D') = 0$ if $\alpha' \geq 0$, and $\mu(\alpha', D') = 1$ if $\alpha' < 0$.

Note that these separating equilibria are essentially equivalent in that the creditor and both types of firm are indiffferent among them. Myers and Majluf identify perhaps the most natural of these equilibria: $(\alpha_H, D_H) = (0, R)$ and $(\alpha_L, D_L) = (\pi_L/R, 0)$, so that the high-profit firm issues debt and the low-profit firm equity.

As is well known, it is common for signaling games to have a plethora of perfect Bayesian equilibria; as we have just shown, our model is no exception. In some models, many of these equilibria are supported by unreasonable beliefs off the equilibrium path. Several recent papers

propose stronger definitions of equilibrium that restrict these out-of-equilibrium beliefs. Cho and Kreps [1987], for instance, propose a refinement which isolates a unique equilibrium in Spence's original education-signaling setting. As we discuss in the conclusion, however, the Cho-Kreps refinement has considerably less power in our model.

In this paper we adopt a stronger refinement based on the definition of neologism-proof equilibrium developed by Farrell [1983] and the definition of perfect sequential equilibrium developed by Grossman and Perry [1986]. The refinement involves what we call a <u>consistent</u> <u>interpretation</u> of a deviation from a perfect Bayesian equilibrium.

Formally, an <u>interpretation</u> of a deviation is a (non-empty) subset of the type space: the creditor observes the deviation and hypothesizes that some member of the specified subset is responsible for the deviation. In this two-type model, the only possible interpretations are $\{\pi_L\}$, $\{\pi_H\}$, and $\{\pi_L, \pi_H\}$. Given an interpretation, the creditor's posterior belief is its prior belief renormalized over the specified subset. Given a posterior belief, sequential rationality determines whether the creditor accepts or rejects the deviation; if the creditor is indifferent, we assume that the deviation is accepted. Thus, given an interpretation of a deviation, each type in the type space can compute its payoff from offering the deviation, and can compare this payoff to its payoff in the perfect Bayesian equilibrium in question. A consistent interpretation is a fixed point of the map described above: a type strictly prefers its payoff from offering the deviation to its equilibrium payoff if and only if it is a member of the hypothesized subset of the type space.

We argue that if a deviation has a consistent interpretation then it

is natural for the creditor to hold the associated posterior belief. By construction, this belief motivates an acceptance decision that induces some types to deviate, thereby destroying the equilibrium in question. We therefore strengthen our definition of equilibrium to require:

> (E3) There cannot exist a deviation with a consistent interpretation.

We call an equilibrium that satisfies (E1)-(E3) a Farrell-Grossman-Perry (FGP) equilibrium.

Interestingly, all the perfect Bayesian equilibria identified above are FGP equilibria. (The proof is simple and unenlightening, so we omit it.) Loosely, this happens because the beliefs we use to support these equilibria are reasonable, in the following sense. A deviation (α',D') with $\alpha' > 0$ is more attractive to the π_L-type than to the π_H-type. This suggests that when $\alpha' > 0$ the creditor's belief should put more weight on π_L than on π_H, as is the case (in the extreme) in our equilibria. The analogous intuition applies when $\alpha' < 0$.

3.   Product-Market Competition under Asymmetric Information

In this section we model the product market and thereby endogenize the gross profits taken as exogenous in the previous section. We first present a general model of product-market competition under asymmetric information. Then we describe two simple examples that differ in a way that is the key to our results.

Let $\pi(q,t)$ be the profit to firm A when its type is $t \varepsilon \{\underline{t},\bar{t}\}$ and it is common knowledge that firm B believes that the probability that $t = \bar{t}$ is $q \varepsilon [0,1]$. (Since we use $\pi(q,t)$ to determine payoffs off the equilibrium path, it is important not to interpret q as the common-knowledge

probability that nature draws t = t̄ for firm A: firm B's belief could be wrong.)

This general model is deliberately vague about the game between firm A and firm B that generates these profits.   This allows our single formulation to cover a broad collection of specific models, including those characterized as "strategic substitutes" and "strategic complements" by Bulow, Geanakoplos, and Klemperer [1985].   The single assumption on $\pi(q,t)$ that we maintain throughout our analysis is that gross profits are sufficient to avoid bankruptcy: (AO) $\pi(q,t) > R$ for all $q\varepsilon[0,1]$ and $t\varepsilon\{\underline{t},\overline{t}\}$.

We simplify some of our characterization results by imposing two additional regularity conditions on $\pi(q,t)$.   The first is that given firm B's belief, firm A prefers that its type be t̄:   (A1) $\pi(q,\overline{t}) > \pi(q,\underline{t})$ for all $q\varepsilon[0,1]$.   In many familiar models, this is merely a definition of the types $\underline{t}$ and $\overline{t}$.   The second is that firm A's profit is monotone in firm B's belief: (A2) $\pi(q,t)$ is strictly monotone in q.   Two cases are possible: (A2a) $\pi_q < 0$, and (A2b) $\pi_q > 0$.   One or the other of these two conditions holds in most applications.

Our main result does not depend on assumptions (A1) or (A2);   see Propositions 1 and 2.   Instead, the result is driven by a feature of product-market competition that we characterize in the two examples below.

Example 1.   Consider an industry with an inverse-demand function given by $P(Q) = a - Q$, where Q is industry supply.   We consider a Cournot duopoly, consisting of one firm, labelled A, that knows the value of the demand intercept, a, and another, labelled B, that initially believes

that $a = \bar{a}$ with probability $\theta\epsilon(0,1)$ and $a = \underline{a}$ with probability $1 - \theta$,
where $\bar{a} > \underline{a}$. Costs are assumed to be zero.

We now compute several profit levels for the informed firm, A, that
arise in the analysis below. These profit levels correspond to different
beliefs held by firm B. If, for instance, the value of the intercept, a,
were common knowledge, then both firms would produce a/3 and earn profit
$(a/3)^2$. We therefore define $\bar{\pi}^s \equiv (\bar{a}/3)^2$ and $\underline{\pi}^s \equiv (\underline{a}/3)^2$, the profit to
firm A when it is common knowledge that $a = \bar{a}$ and $a = \underline{a}$, respectively.
We use the superscript "s" to denote that this information structure
arises when firm B observes the result of a separating equilibrium of the
game between firm A and the capital market.

When firm B holds the prior belief that $a = \bar{a}$ with probability θ,
the firms play a Cournot game of incomplete information. The equilibrium
strategies are

$$q_B = [\theta\bar{a}+(1-\theta)\underline{a}]/3,$$

$$\underline{q}_A = [2\underline{a}-\theta\Delta]/6, \quad \text{and}$$

$$\bar{q}_A = [2\bar{a}+(1-\theta)\Delta]/6,$$

where q_B is firm B's output, $\underline{q}_A$ (resp. $\bar{q}_A$) is firm A's output when $a = \underline{a}$
(resp. $\bar{a}$), and $\Delta \equiv \bar{a} - \underline{a}$. These strategies result in profits for firm A
of

$$\underline{\pi}^P \equiv [2\underline{a}-\theta\Delta]^2/36 \quad \text{and} \quad \bar{\pi}^P \equiv \left[2\bar{a}+(1-\theta)\Delta\right]^2/36$$

when $a = \underline{a}$ and $a = \bar{a}$, respectively. We use the superscript "p" to denote

that this information structure arises when firm B observes the result of

a pooling equilibrium of the game between firm A and the capital market.

Finally, we need to determine firm A's profits when firm B holds

mistaken beliefs about the value of the intercept. (These profits never

arise in equilibrium, but are relevant off the equilibrium path.) If

firm B believes that $a = \underline{a}$ when in fact $a = \bar{a}$, then firm A's output is

$(2\bar{a}+\Delta)/6$, and its resulting profit is $\bar{\pi}' \equiv [2\bar{a}+\Delta]^2/36$. Similarly, if

firm B believes that $a = \bar{a}$ when in fact $a = \underline{a}$, then firm A's output is

$(2\underline{a}-\Delta)/6$. We assume that $\Delta < 2\underline{a}$, so that this quantity is positive. The

associated profit is then $\underline{\pi}' \equiv [2\underline{a}-\Delta]^2/36$. We use the prime to denote

that these profits arise off the equilibrium path.

It is straightforward to show that in this example

$$\theta\bar{\pi}^p + (1-\theta)\underline{\pi}^p > \theta\bar{\pi}^s + (1-\theta)\underline{\pi}^s.$$

That is, ex-ante expected profits to firm A are higher when firm B is

uninformed than when firm B is informed. These information structures

correspond, respectively, to pooling and separating equilibria in the

capital market. This inequality is an important feature of this example.

Our next example shows that this inequality can be reversed, and Section

5 shows that whether this inequality holds or is reversed completely

determines the form of equilibrium in our two-audience signaling model.

Example 2. Suppose now that firm B is a potential entrant into the

industry described in Example 1. Naturally, firm B bases its entry

decision on its beliefs about the demand intercept. If firm B enters,

then Cournot competition ensues, exactly as described above. The fixed cost of entry is F_B. Assume that firm B can finance the fixed entry cost with internal funds.

If firm B enters and believes the demand intercept is $\bar{a}$ (resp. $\underline{a}$), its gross profit is $(\bar{a}/3)^2$ (resp. $(\underline{a}/3)^2$). If firm B enters and believes $a = \bar{a}$ with probability θ, its expected gross profit is $(\tilde{a}/3)^2$, where $\tilde{a} \equiv \theta\bar{a} + (1-\theta)\underline{a}$. Assume that

$$(\bar{a}/3)^2 > (\tilde{a}/3)^2 > F_B > (\underline{a}/3)^2.$$

Then if firm B believes demand is low, it will not enter, while if firm B believes demand is high or believes that it is high with probability θ, it will enter.

If there is no entry, firm A earns monopoly profits $(\bar{a}/2)^2$ when $a = \bar{a}$, and $(\underline{a}/2)^2$ when $a = \underline{a}$. The values $\bar{\pi}^s$, $\bar{\pi}^p$, $\underline{\pi}^p$, and $\underline{\pi}'$ derived in Example 1 are the same here, but $\underline{\pi}^s$ is now $(\underline{a}/2)^2$ and $\bar{\pi}'$ is now $(\bar{a}/2)^2$, because firm B does not enter when it believes $a = \underline{a}$. It is straight-forward to show that

$$\theta\bar{\pi}^s + (1-\theta)\underline{\pi}^s > \theta\bar{\pi}^p + (1-\theta)\underline{\pi}^p$$

if and only if $\underline{a}^2 > \theta\Delta^2$. As in the previous example, we assume that $\Delta > 2\underline{a}$, so that $\underline{\pi}' > 0$. Thus $\underline{a}^2 > \Delta^2/4$, so $\theta < 1/4$ guarantees that $\underline{a}^2 > \theta\Delta^2$, as required.

4. A Two-Audience Signaling Model

This section integrates the capital-market signaling described in Section 2 with the product-market competition described in Section 3. The timing of the game is as follows. First, firm A learns its type, t. It is common knowledge that the state is drawn from $\{\underline{t},\bar{t}\}$, with probability $\theta\varepsilon(0,1)$ that $t = \bar{t}$. Next, firm A offers the creditor a financing contract, which the creditor either accepts or rejects. If the creditor rejects the contract then the game ends; if the creditor accepts the contract then firm B observes the accepted contract, and the two firms play the product-market game of incomplete information described above. In what follows we (loosely) refer to this product-market game of incomplete information as the post-contract subgame.

As in Section 2, we restrict attention to linear contracts, (α,D). We explain in the Conclusion that, because our model has only two types, restricting attention to linear contracts does not entail any significant loss of generality in our two-audience signaling model.

In this game, a strategy for firm A is a function from its type space into both the contract space and firm A's strategy space for the post-contract subgame.[2] A strategy for the creditor is again a function from the contract space into an acceptance decision. Finally, a strategy for firm B is a function from the contract space into firm B's strategy space for the post-contract subgame.

We also need to define beliefs held by the uninformed players after firm A offers the contract (α,D): let $\mu(\alpha,D)$ be the creditor's assessment of the probability that $t = \bar{t}$, and let $\eta(\alpha,D)$ be the analogous assessment for firm B.

A perfect Bayesian equilibrium (PBE) in this game is a triple of strategies and a pair of belief functions satisfying two conditions:

(E1') (i) Given its type, the creditor's strategy, and firm B's strategy, firm A's strategy maximizes its expected payoff.

(ii) Given a contract offered by firm A, firm B's strategy, and the creditor's belief, the creditor's strategy maximizes its expected payoff.

(iii) Given a contract offered by firm A and accepted by the creditor, and given firm B's belief, firm B's strategy maximizes its expected payoff.

(E2') When possible, the beliefs $\mu(\alpha, D)$ and $\eta(\alpha, D)$ are derived from Bayes' rule and the players' strategies.

As discussed in Section 2, we consider a further restriction on equilibrium, based on consistent interpretations of deviations. The definition of a consistent interpretation must be extended slightly because there are now two uninformed players. As before, an interpretation of a deviation is a (non-empty) subset of the type space of firm A. In our two-type model, only three interpretations are possible: $\{\underline{t}\}$, $\{\bar{t}\}$, and $\{\underline{t}, \bar{t}\}$. We say that an uninformed player believes an interpretation if its posterior belief after observing the deviation is its prior belief renormalized over the set of types specified in the interpretation. Suppose it is common knowledge that firm B believes a given interpretation. Compute the gross payoff for each type of firm A in the post-contract subgame. Suppose also that the creditor believes the given interpretation. Given an interpretation, the creditor's posterior belief is its prior belief renormalized over the specified subset. Given a posterior belief, sequential rationality determines whether the creditor

accepts or rejects the deviation; if the creditor is indifferent, we assume that the deviation is accepted. Thus, given an interpretation of a deviation, each type in the type space can compute its payoff from offering the deviation, and can compare this payoff to its payoff in the perfect Bayesian equilibrium in question. A consistent interpretation is a fixed point of the map described above: a type strictly prefers its payoff from offering the deviation to its equilibrium payoff if and only if it is a member of the hypothesized subset of the type space.

Less formally, in a multi-audience signaling game, a deviation with a consistent interpretation has the property that if all the uninformed players believe the interpretation, then the hypothesized types of the informed player have a (strict) incentive to deviate. Thus, we require the creditor's and firm B's beliefs off the equilibrium path to be identical following deviations with consistent interpretations, but not otherwise.[3]

As in Section 2, only three consistent interpretations are possible in our two-type model. In this section, we refer to them as $\bar{t}$-separating, $\underline{t}$-separating, and pooling consistent interpretations. If a deviation has any of these consistent interpretations, the deviation breaks the equilibrium. Thus, an FGP equilibrium satisfies (E1'), (E2'), and (E3'):

    (E3') There cannot exist a deviation with a consistent interpreta-
            tion.

We reiterate that the consistent interpretation mentioned in (E3') differs from that mentioned in (E3), because the former requires an uninformed player to hold beliefs about what another uninformed player believes.

5. Analysis

This section characterizes equilibrium in our model. The main result is that the FGP equilibria are of the form (separating or pooling) that maximizes ex-ante expected gross profit for firm A. Formally, if

$$(*) \qquad \theta\bar{\pi}^P + (1-\theta)\underline{\pi}^P \geq \theta\bar{\pi}^S + (1-\theta)\underline{\pi}^S$$

then pooling maximizes ex-ante expected profit, while if (*) fails or holds with equality then separation maximizes ex-ante expected profit. Recall that Examples 1 and 2 in Section 3 establish that both cases are possible.

To prove this result, we begin by establishing a pair of necessary conditions. First, we show that a necessary condition for the existence of a pooling FGP equilibrium is that ex-ante expected profit for firm A is weakly greater under pooling than under separation (i.e., that (*) holds). Then we show that a necessary condition for the existence of a separating FGP equilibrium is that (*) fails or holds with equality.

Consider first a pooling FGP equilibrium in which both types of firm A offer the contract (α, D). If the creditor earns positive expected profit on this contract, then the deviation (α', D'), with D' slightly less than D, has a pooling consistent interpretation: if both the creditor and firm B believe that both types would offer this deviation, so that $\mu(\alpha', D') = \eta(\alpha', D') = \theta$, then both types indeed choose to deviate. This deviation leaves product-market payoffs unchanged while reducing financing costs for both types. Therefore, the creditor's expected profit is zero in a pooling FGP equilibrium:

(1) $\qquad \alpha\widetilde{\pi}^P + (1-\alpha)D = R,$

where $\widetilde{\pi}^P \equiv \theta\overline{\pi}^P + (1-\theta)\underline{\pi}^P.$[4]

We now consider the conditions under which a deviation (α',D') has either a $\overline{t}$-separating or a $\underline{t}$-separating consistent interpretation. These conditions establish a necessary condition for a pooling FGP equilibrium to exist: (*) must hold.

Consider first a deviation (α',D') that has a $\overline{t}$-separating consistent interpretation. This requires:

(2) $\qquad \alpha'\overline{\pi}^S + (1-\alpha')D' \geq R,$

(3) $\qquad (1-\alpha')(\overline{\pi}^S-D') > (1-\alpha)(\overline{\pi}^P-D), \quad$ and

(4) $\qquad (1-\alpha)(\underline{\pi}^P-D) \geq (1-\alpha')(\underline{\pi}'-D').$

Inequality (2) guarantees that the creditor earns non-negative profit; (3) and (4) ensure that only the $\overline{t}$-type firm accepts the new contract. (Note that the $\underline{t}$-type's limited-liability constraint does not affect whether (4) holds because this constraint can at most make the righthand side zero, while individual rationality insures that the equilibrium payoff on the lefthand side must be non-negative. This argument holds throughout the analysis.)

To establish the necessary condition, we choose to consider only those deviations in which (2) holds with equality. Substituting this equation and (1) into (3) yields

$$(5) \qquad \alpha(1-\theta)(\bar{\pi}^P - \underline{\pi}^P) > \bar{\pi}^P - \bar{\pi}^S,$$

and substituting the same equalities into (4) yields

$$(6) \qquad \alpha'\left[\bar{\pi}^S - \underline{\pi}'\right] \leq \alpha\theta(\bar{\pi}^P - \underline{\pi}^P) + \left[\underline{\pi}^P - \underline{\pi}'\right].$$

Given α, there exists an α' satisfying (6). (Recall that we do not restrict α' to $[0,1]$.) Therefore, there exists a deviation with a $\bar{t}$-separating consistent interpretation if (5) holds.

Consider next a deviation (α',D') that has a $\underline{t}$-separating consistent interpretation. This requires:

$$(7) \qquad \alpha'\underline{\pi}^S + (1-\alpha')D' \geq R,$$

$$(8) \qquad (1-\alpha')(\underline{\pi}^S - D') > (1-\alpha)(\underline{\pi}^P - D), \quad \text{and}$$

$$(9) \qquad (1-\alpha)(\bar{\pi}^P - D) \geq (1-\alpha')(\bar{\pi}' - D').$$

These inequalities are analogous to (2)-(4). As before, we consider only those deviations in which (7) holds with equality. Substituting this equation and (1) into (8) yields

$$(10) \qquad \alpha\theta(\bar{\pi}^P - \underline{\pi}^P) < \underline{\pi}^S - \underline{\pi}^P,$$

and substituting the same equalities into (9) yields

$$(11) \qquad \alpha'\left[\bar{\pi}'-\underline{\pi}^S\right] \geq \alpha(1-\theta)(\bar{\pi}^P-\underline{\pi}^P) + \left[\bar{\pi}'-\bar{\pi}^P\right].$$

As before, given α, there exists an α' satisfying (11). Therefore, there exists a deviation with a $\underline{t}$-separating consistent interpretation if (10) holds.

These two potential deviations establish that a necessary condition for a pooling FGP equilibrium to exist is that (5) and (10) fail:

$$(12) \qquad \theta(\bar{\pi}^P-\bar{\pi}^S) \geq \alpha\theta(1-\theta)(\bar{\pi}^P-\underline{\pi}^P) \geq (1-\theta)(\underline{\pi}^S-\underline{\pi}^P),$$

which implies (*). We record this result as Proposition 1.

<u>Proposition 1</u>. A necessary condition for the existence of a pooling FGP equilibrium is that ex-ante expected profit for firm A is weakly greater under pooling than under separation: $\theta\bar{\pi}^P + (1-\theta)\underline{\pi}^P \geq \theta\bar{\pi}^S + (1-\theta)\underline{\pi}^S$. Moreover, if this inequality holds then α must lie in the interval determined by (12).

This result does <u>not</u> require assumptions (A1) or (A2). Imposing various combinations of these assumptions, however, allows us to interpret the pooling FGP equilibria. Under (A1) and (A2a), for instance, $\bar{\pi}^P > \underline{\pi}^P$ and $\underline{\pi}^S > \underline{\pi}^P$, in which case the second inequality in (12) implies that any pooling FGP equilibrium must have $\alpha > 0$. In this case, (1) implies that the $\bar{t}$-type subsidizes the $\underline{t}$-type in the capital market: $\bar{t}(\underline{t})$ has financing costs greater (less) than R. In the product market, on the other hand, $\bar{t}(\underline{t})$ has higher (lower) gross profit when capital-market equilibrium is pooling than when it is separating: $\bar{\pi}^P \equiv \pi(\theta,\bar{t}) >$

$\bar{\pi}^S \equiv \pi(1,\bar{t})$ but $\underline{\pi}^S \equiv \pi(0,\underline{t}) > \underline{\pi}^P \equiv \pi(\theta,\underline{t})$ because $\pi_q < 0$. This proves our claim that, given mild regularity conditions, reasonable pooling equilibria must be balanced.

When (A1) and (A2b) hold, $\bar{\pi}^P > \underline{\pi}^P$ and $\bar{\pi}^S > \bar{\pi}^P$, in which case the first inequality in (12) implies that any pooling FGP equilibrium must have $\bar{\alpha} < 0$. Now, the $\underline{t}$-type subsidizes the $\bar{t}$-type in the capital market. And in the product market it is $\bar{t}$ that suffers in a pooling equilibrium. Again, reasonable pooling equilibria are balanced.

Finally, (12) implies that either (A2a) or (A2b) is sufficient to preclude pooling at debt.

Corollary. If $\pi(q,t)$ is strictly monotonic in q, then there cannot exist a pooling FGP equilibrium in which both firms offer a debt contract, (0,D).

This result is in stark contrast to the analogous result in Section 2: when the analysis ignores competition in the product market, so that firm A's gross profits are exogenous, the only pooling equilibrium is the zero-profit debt contract (0,R).

We now consider a separating FGP equilibrium $\{(\underline{\alpha},\underline{D}),(\bar{\alpha},\bar{D})\}$. It must satisfy the following incentive-compatibility conditions.

$$(13) \qquad (1-\bar{\alpha})(\bar{\pi}^S-\bar{D}) \geq (1-\underline{\alpha})[\bar{\pi}'-\underline{D}],$$

$$(14) \qquad (1-\underline{\alpha})(\underline{\pi}^S-\underline{D}) \geq (1-\bar{\alpha})[\underline{\pi}'-\underline{D}].$$

In this notation, $(\underline{\alpha},\underline{D})$ is the contract offered by firm A when $t = \underline{t}$, and $(\bar{\alpha},\bar{D})$ is offered when $t = \bar{t}$. These contracts must yield non-negative profits for the creditor:

$$(15) \qquad \bar{\alpha}\bar{\pi}^S + (1-\bar{\alpha})\bar{D} \geq R, \quad \text{and}$$

$$(16) \qquad \underline{\alpha}\underline{\pi}^S + (1-\underline{\alpha})\underline{D} \geq R.$$

Consider a deviation (α',D') that has a pooling consistent interpretation. This requires:

$$(17) \qquad (1-\alpha')(\tilde{\pi}^P-D') > (1-\bar{\alpha})(\bar{\pi}^S-\bar{D}), \quad \text{and}$$

$$(18) \qquad (1-\alpha')(\tilde{\pi}^P-D') > (1-\underline{\alpha})(\underline{\pi}^S-\underline{D}).$$

We consider only deviations that earn zero expected profit for the creditor:

$$(19) \qquad \alpha'\tilde{\pi}^P + (1-\alpha')D' = R,$$

where $\tilde{\pi}^P \equiv \theta\bar{\pi}^P + (1-\theta)\underline{\pi}^P$, as above. Substituting (19) into (17) and (18) yields

$$(20) \qquad (1-\theta)\left\{\underline{\pi}^S-\underline{\pi}^P+\left(R-\underline{\alpha}\underline{\pi}^S-(1-\underline{\alpha})\underline{D}\right)\right\} < \theta(1-\theta)(\bar{\pi}^P-\underline{\pi}^P)\alpha'$$
$$< \theta\left\{\bar{\pi}^P-\bar{\pi}^S+\left(\bar{\alpha}\bar{\pi}^S+(1-\bar{\alpha})\bar{D}-R\right)\right\}.$$

Thus, there exists a deviation if (20) holds. This establishes that a

necessary condition for a separating FGP equilibrium to exist is that the

lefthand side of (20) weakly exceed the righthand side. After applying

(15) and (16), this becomes

$$(21) \qquad (1-\theta)\left[\underline{\pi}^S - \underline{\pi}^P\right] \geq \theta\left[\bar{\pi}^P - \bar{\pi}^S\right],$$

so we have established that a necessary condition for a separating FGP

equilibrium to exist is that (*) must fail or hold with equality. We

record this result as Proposition 2.

Proposition 2. A necessary condition for the existence of a separating

FGP equilibrium is that ex-ante expected profit for firm A is weakly

greater under separation than under pooling: $\theta\bar{\pi}^S + (1-\theta)\underline{\pi}^S \geq$

$\theta\bar{\pi}^P + (1-\theta)\underline{\pi}^P$.

Propositions 1 and 2 establish necessary conditions for the

existence of pooling and separating FGP equilibria; they do not, however,

establish that such equilibria exist. We now prove existence by estab-

lishing that the contracts in question can be supported as perfect

Bayesian equilibria.

The following pooling perfect Bayesian equilibria exist in our

model. (Proofs of Claims 1 and 2 and Propositions 3 and 4 are collected

in the Appendix.) Note that we have not attempted a complete character-

ization of the perfect Bayesian equilibria.

<u>Claim 1</u>: Given (A1), the contract (α, D) can be supported as a pooling PBE if $\alpha\left[\theta\bar{\pi}^P + (1-\theta)\underline{\pi}^P\right] + (1-\alpha)D = R$ and: (i) given (A2a),

$$(22) \qquad \theta(\bar{\pi}^P - \bar{\pi}^S) \geq \alpha\theta(1-\theta)(\bar{\pi}^P - \underline{\pi}^P) \geq (1-\theta)\left[\underline{\pi}' - \underline{\pi}^P\right];$$

or (ii) given (A2b),

$$(23) \qquad \theta\left[\bar{\pi}^P - \bar{\pi}'\right] \geq \alpha\theta(1-\theta)(\bar{\pi}^P - \underline{\pi}^P) \geq (1-\theta)(\underline{\pi}^S - \underline{\pi}^P).$$

The intuition for this claim is closely related to the discussion following Proposition 1 that proves that reasonable pooling equilibria must be balanced. In that discussion we established that if (A2a) holds then $\alpha > 0$ is necessary to balance the product- and capital-market effects. It is not the case, however, that pooling PBEs need be balanced: the second inequality in (22) establishes that, given (A2a), α can be negative in a pooling PBE. In this case, the $\underline{t}$-type firm actually subsidizes the $\bar{t}$-type firm in the capital market, even though the $\underline{t}$-type firm would prefer separation to pooling for its product-market effects. This is possible because the creditor and firm B can (in a PBE, at least) believe the worst about firm A. There is a limit to how bad this can be for the $\underline{t}$-type, however, and this limit yields the (negative) lower bound on α in (22).

The intuition for (23) is analogous. If (A2b) holds then $\alpha < 0$ is necessary to balance the product- and capital-market effects, but the first inequality in (23) shows that $\alpha > 0$ is possible in a pooling PBE, which implies that the $\bar{t}$-type subsidizes the $\underline{t}$-type in the capital

market, even though the $\bar{t}$-type prefers separation in the product market. As above, such an equilibrium is supported by extreme beliefs off the equilibrium path.

Perfect Bayesian equilibria that depend on implausible beliefs do not survive the FGP refinement. When (A2a) holds, a comparison of (12) and (22) clarifies the discipline that the FGP refinement imposes on beliefs: the only difference is that $\underline{\pi}^s$ in (12) becomes $\underline{\pi}'$ in (22); in the latter, firm B interprets all deviations as having come from the $\bar{t}$-type, even though this is not a consistent interpretation. Similarly, when (A2b) holds, the only difference between (12) and (23) is that $\bar{\pi}^s$ in (12) becomes $\bar{\pi}'$ in (23). Once again, the latter relies on an inconsistent interpretation of a deviation, (this time involving the $\underline{t}$-type), even though a consistent interpretation exists.

We summarize this informal discussion of pooling FGP equilibria with the following complete characterization.

<u>Proposition 3.</u> Given (A1), (A2), and (*), the contract (α, D) can be supported as a pooling FGP equilibrium if and only if α satisfies (12) and (α, D) satisfies the zero-profit condition (1).

We now turn to a discussion of separating equilibria. The following separating perfect Bayesian equilibria exist in our model. As in Claim 1, we have not attempted a complete characterization.

<u>Claim 2.</u> Given (A1) and (A2), the contracts $(\underline{\alpha}, \underline{D})$ and $(\bar{\alpha}, \bar{D})$ can be supported as a separating PBE if the non-negative profit constraints (15) and (16) hold with equality,

$$(24) \quad \underline{\alpha} \geq \frac{\bar{\pi}' - \bar{\pi}^s}{\bar{\pi}' - \underline{\pi}^s}, \quad \text{and}$$

$$(25) \quad \bar{\alpha} \leq \frac{\underline{\pi}^s - \underline{\pi}'}{\bar{\pi}^s - \underline{\pi}'}.$$

These bounds on the equity components follow directly from the incentive-compatibility constraints (13) and (14) after substituting for the debt components using the zero-profit conditions. Because both types of firm A pay financing costs of R in these equilibria, they can gain from deviating only if the deviation is favorably interpreted by either the creditor or firm B. The proof of Claim 2 relies on beliefs for the uninformed players that avoid such favorable interpretations, but Proposition 4 indicates that such interpretations sometimes are the only reasonable ones.

Proposition 4. Given (A1) and (A2), the contracts $(\underline{\alpha}, \underline{D})$ and $(\bar{\alpha}, \bar{D})$ specified in Claim 2 can be supported as FGP equilibria if and only if (*) fails or holds with equality.

We can relate (24) and (25) to the Myers-Majluf intuition that low-profit firms issue more equity: if $\pi(q,t)$ also satisfies $\pi_q(q,\bar{t}) - \pi_q(q,\underline{t}) < 0$ for all $q \in [0,1]$, then adding the incentive-compatibility constraints (13) and (14) implies that $\underline{\alpha} > \bar{\alpha}$ in all separating equilibria.

6. Conclusion

This paper introduces the notion of a two-audience signaling model. Although such models have received little attention in the literature, we feel that a rich collection of potential applications exists. First, the gross profit function $\pi(q,t)$ introduced in Section 4 is general enough that the second uninformed audience in our model can be reinterpreted (without any changes in the model) as a labor union or a regulator rather than a product-market competitor. Second, moving away from our capital-market signaling context, a model of product-market signaling (such as the limit pricing analyzed by Milgrom and Roberts [1982] or the predation analyzed by Saloner [1986]) could be enriched to include a second audience, such as the capital market or a labor union. Finally, an analogous problem arises when there is two-sided incomplete information, as when an informed agent signals to a single audience with its own private information. Some dynamic analyses of this kind have already appeared in the bargaining literature; see Cramton [1986] and Chatterjee and Samuelson [1987].

The rest of this conclusion considers some of the issues raised but incompletely treated in the body of the paper. We discuss, in turn, game-theoretic modeling issues (including our choice of a refinement concept), other modeling issues (including our focus on linear financing contracts), and various possible extensions of the analysis (including a model of an initial public offering).

6.1 Game-Theoretic Modeling Issues

This paper models information transmission as a signaling game: the informed player moves before the uninformed players move. In this

setting, our main result follows from the refinement concept we impose on equilibrium in the signaling game. This sub-section considers two modifications of our model: (1) alternative refinement concepts in signaling games, and (2) alternative games of information transmission, in which an uninformed player moves first.

The refinement we impose, that no deviation has a consistent interpretation, involves a strong restriction on beliefs off the equilibrium path. As mentioned in Section 2, Cho and Kreps (CK) propose a weaker restriction: out-of-equilibrium beliefs should put no weight on types that have no incentive to deviate <u>no matter what</u> the creditor believes about the set of types who deviated. The weaker CK refinement has an interesting effect in our model: Proposition 1 continues to hold, but Proposition 2 does not. To see why this is so, recall that Proposition 1 depends on $\bar{t}$-separating and $\underline{t}$-separating consistent interpretations of deviations from a putative pooling equilibrium. Because we do not require $\alpha\varepsilon[0,1]$ and $D \geq 0$, there exist analogous deviations that satisfy the stronger CK test: no matter what the uninformed players infer from the deviation, one type (say, $\underline{t}$) has no incentive to offer such a deviation, so the uninformed infer that $\bar{t}$ offered the deviation, and this induces $\bar{t}$ to deviate. Proposition 2, however, depends on a pooling consistent interpretation of a deviation from a putative separating equilibrium. The essence of this interpretation is that <u>both</u> types prefer to deviate, and so it seems natural that the belief off the equilibrium path should be the prior belief, as is the case under the FGP refinement. The CK refinement, however, has force only when some types certainly would not deviate. We think that the pooling consistent interpretation is natural in our model, so we are pleased with the effect of

the FGP refinement. More generally, we would like a refinement concept to speak to such pooling deviations, and the CK refinement (as well as its stronger cousins, such as "never a weak best-response") can never do this.

We turn now to an alternative game of information transmission, in which an uninformed player moves first. This timing mimics the Rothschild-Stiglitz [1976] and Wilson [1980] (RSW) models of insurance markets. In a pure capital-market model, like that in Section 2, the timing is now as follows: Each of two competing creditors simultaneously offers a set of financing contracts to the informed firm. The informed firm then accepts at most one of the offered contracts.

It is straightforward to show that the subgame-perfect equilibrium contracts accepted in this RSW game are exactly those accepted in equilibria that survive the FGP refinement in the signaling game in Section 2 (and therefore, by our results in that section, exactly those accepted in perfect Bayesian equilibria of the signaling game). This result is closely related to recent work in more general settings on the relationship between competitive equilibria in RSW games and equilibria that survive refinements in analogous signaling games.[5]

The RSW analog of the model we analyze in Sections 4 and 5 is complicated by the presence of the uninformed firm B. The timing we have in mind is that firm B observes the outcome of the RSW game described above, and then firms A and B play the post-contract subgame described in Section 3. Thus, the entire game is composed of: first, an RSW stage involving the creditors and firm A; and second, a signaling stage involving firm A and firm B, in which the set of signals firm A is able to send is endogenously determined by the creditors' moves in the RSW stage.

As in all signaling games, the uninformed firm B's out-of-equilibrium beliefs substantially affect the character of equilibrium. Thus, despite the apparent power of the RSW timing of the first stage, it turns out that the signaling timing of the second stage makes it necessary to apply a refinement concept to the game as a whole to eliminate implausible equilibria. In an earlier phase of this work, we analyzed an FGP-like refinement in the game just described. (We slightly modified the definition of the refinement to accommodate the timing described above.) The equilibrium contracts that survived this refinement in the RSW game were exactly the FGP equilibria in the signaling game analyzed in Sections 4 and 5. It remains an open question whether a weaker refinement applied to the RSW game might, because of the apparent power of the RSW timing, still yield the same equivalence result.

6.2 Other Modeling Issues

Our analysis assumes that any linear financing contract is feasible. Of course, equity components (α) outside [0,1] and debt components (D) less than zero seem rare in the world. A preliminary analysis suggests that requiring $\alpha \epsilon$ [0,1] and $D \geq 0$ considerably complicates the results. For instance, pooling equilibrium may not exist when the capital-market payoffs are small relative to the product-market payoffs (i.e., R is small relative to $\pi(q,t)$). A loose statement of the intuition is as follows. Consider a pooling FGP equilibrium under (A2a): (12) requires $\alpha \geq (\underline{\pi}^S - \underline{\pi}^P)/[\theta(\bar{\pi}^P - \underline{\pi}^P)]$, which is strictly positive; the zero-profit constraint, on the other hand, requires $D = (R - \alpha\tilde{\pi})/(1-\alpha)$, which may be negative if R is small relative to $\tilde{\pi}$ and α is large. This intuition is

loose because the deviations considered in the derivation of (12) allow α outside [0,1] and D < 0. In fact, pooling equilibria can exist in the model with a restricted contract space even when R is arbitrarily small, provided something like the sorting condition familiar from one-audience signaling models holds. Independent of whether pooling equilibria exist, pooling deviations that break putative separating equilibria may still exist. Under certain conditions, no equilibrium exists in the model with a restricted contract space. As is well known, similar non-existence results arise in the original applications of the FGP refinement, as well as in some versions of the Rothschild-Stiglitz-Wilson game.

In one sense, then, our feasible set of financing contracts is too large. In another sense, however, the set of linear contracts we consider may seem too small: non-linear contracts, such as step functions, can be important in models that involve a moving support (i.e., some profit levels that are feasible for one type are infeasible for another). In our two-type model, however, no great damage is done by ignoring these contracts. This point is clear in the simple model in Section 2, but also true in the two-audience model in Sections 4 and 5, because of a combination of arguments. First, the set of payoffs to perfect Bayesian equilibria in our model (weakly) includes the set of payoffs to perfect Bayesian equilibria in the analogous model with general contracts. Second, if a general contract is a deviation with a consistent interpretation then there exists a linear contract that is a deviation with the same consistent interpretation. (These two arguments imply that the set of payoffs to FGP equilibria in our model (weakly) includes the set of payoffs to FGP equilibria in the analogous model with

general contracts.) Finally, our main result takes the form of necessary conditions; see Proposition 1, its Corollary, and Proposition 2. Together, these arguments imply that our main result holds for the model with general contracts. The characterization results in Proposition 3 and 4, however, may not.

This argument is, of course, very sensitive to the number of types in the model. If there are more than two types, non-linear contracts could have a significant effect. On the other hand, we suspect that in a richer model, involving ex-post uncertainty as well as limited liability, these contracts will not play a crucial role.

Finally, we conjecture that our main result may generalize in an interesting way. Consider a model with any finite number of types. This finite type set can be partitioned in a finite number of ways. For the case of two types there are two partitions: pooling and separating. For three types there are five: pooling, separating, and three partial-pooling partitions. Assuming a pure-strategy equilibrium in the capital market, these partitions represent firm B's information structure, and one of them maximizes the ex-ante expected product-market profit for firm A. It seems plausible that the hypothesized subsets of types involved in the FGP refinement may fit naturally with these partitions of the type space. If so, our main result---that the form of equilibrium maximizes firm A's ex-ante expected payoff---may persist.

6.3 Extensions

Our model assumes that firm A's capital structure is observed by the uninformed firm B. This is the correct assumption if firm A is a public corporation. If firm A is not publicly traded, however, then it may be

able to conceal its financial structure from product-market competitors.

Two interesting possibilities arise in this setting. First, while there are no disclosure requirements for a privately held firm, such a firm can choose to reveal its capital structure to product-market competitors, and this choice may act as a signal. And second, the firm may have a choice about whether to be private, and this choice may also be a signal, especially if the cost of capital differs for publicly and privately held firms. A further consequence of going public is that disclosure laws mandate that information other than capital structure be revealed.

Our model also could be extended to allow the possibility of bankruptcy. Brander and Lewis show that bankruptcy may cause the objectives of shareholders and debtholders to diverge, and this may affect the firm's strategic behavior in the product market. The extension we have in mind abstracts from these considerations.

Gertner shows that limited liability and ex-post uncertainty can combine to make pooling at (risky) debt an equilibrium. In such an equilibrium, high-profit firms susidize low-profit firms in the capital market, because the latter face a higher probability of bankruptcy. In a pooling FGP equilibrium in our model, we need subsidization in the capital market to balance the effects of the product market. In particular, under assumption (A2a), a positive equity component ($\alpha > 0$) is required to achieve this balance, and does so by forcing the high-profit firm to subsidize the low-profit firm in the capital market. The subsidization in Gertner's model suggests that a pure-debt contract could achieve this balance in our model if debt were risky.

Appendix

Claim 1. Given (A1), the contract (α, D) can be supported as a pooling PBE if $\alpha\left[\theta\bar{\pi}^P + (1-\theta)\underline{\pi}^P\right] + (1-\alpha)D = R$ and: (i) given (A2a),

$$(22) \qquad \theta(\bar{\pi}^P - \bar{\pi}^S) \geq \alpha\theta(1-\theta)(\bar{\pi}^P - \underline{\pi}^P) \geq (1-\theta)\left[\underline{\pi}' - \underline{\pi}^P\right];$$

(ii) given (A2b),

$$(23) \qquad \theta\left[\bar{\pi}^P - \bar{\pi}'\right] \geq \alpha\theta(1-\theta)(\bar{\pi}^P - \underline{\pi}^P) \geq (1-\theta)(\underline{\pi}^S - \underline{\pi}^P).$$

Proof of Claim 1. Consider a deviation (α', D'). The creditor accepts this deviation if

$$(A.1) \qquad \alpha'\left[\mu\pi(\eta, \bar{t}) + (1-\mu)\pi(\eta, \underline{t})\right] + (1-\alpha')D' \geq R.$$

Even if the creditor will accept the deviation, the $\bar{t}$-type does not have a strict incentive to deviate if

$$(A.2) \qquad (1-\alpha')\left[\pi(\eta, \bar{t}) - D'\right] \leq (1-\alpha)[\bar{\pi}^P - D].$$

Similarly, the $\underline{t}$-type does not have a strict incentive to deviate if

$$(A.3) \qquad (1-\alpha')\left[\pi(\mu, \underline{t}) - D'\right] \leq (1-\alpha)[\underline{\pi}^P - D].$$

We now exhibit belief functions $\mu(\alpha', D')$ and $\eta(\alpha', D')$ such that

neither type has a strict incentive to offer any deviation. Set

$$\mu(\alpha',D') = \begin{cases} 0 & \text{if} \quad \alpha' \geq 0, \\ 1 & \text{if} \quad \alpha' < 0. \end{cases}$$

Then for $\alpha' \geq 0$, (A.1) becomes

(A.4) $\alpha'\pi(\eta,\underline{t}) + (1-\alpha')D' \geq R,$

and for $\alpha' < 0$, (A.1) becomes

(A.5) $\alpha'\pi(\eta,\overline{t}) + (1-\alpha')D' \geq R.$

Using the zero-profit condition (1), (A.2) holds if

(A.6) $\overline{\pi}^P - \pi(\eta,\overline{t}) - \alpha(1-\theta)(\overline{\pi}^P-\underline{\pi}^P) \geq R - \left[\alpha'\pi(\eta,\overline{t})+(1-\alpha')D'\right],$

and (A.3) holds if

(A.7) $\underline{\pi}^P - \pi(\eta,\underline{t}) + \alpha\theta(\overline{\pi}^P-\underline{\pi}^P) \geq R - \left[\alpha'\pi(\eta,\underline{t})+(1-\alpha')D'\right].$

Note that if $\alpha' \geq 0$ then (A.4) and assumption (A1) imply that the righthand sides of (A.6) and (A.7) are non-positive. Similarly, if $\alpha' < 0$ then (A.5) and (A1) imply that these quantities are non-positive. Thus, it suffices to show that

(A.8) $\overline{\pi}^P - \pi(\eta,\overline{t}) - \alpha(1-\theta)(\overline{\pi}^P-\underline{\pi}^P) \geq 0,$ and

(A.9) $\quad \underline{\pi}^P - \pi(\eta,\underline{t}) + \alpha\theta(\bar{\pi}^P - \underline{\pi}^P) \geq 0.$

We now consider the two cases (A2a) and (A2b) given in (i) and (ii) in the claim. First, suppose (A2a) holds. Set $\eta = 1$. Suppose that $\alpha \geq 0$. Then (A.9) clearly holds, and (A.8) yields the first inequality in (22). Now suppose that $\alpha < 0$. Then (A.8) clearly holds and (A.9) yields the second inequality in (22).

Now suppose (A2b) holds. Set $\eta = 0$. Suppose that $\alpha \geq 0$. Then (A.9) clearly holds, and (A.8) yields the first inequality in (23). Now suppose that $\alpha < 0$. Then (A.8) clearly holds, and (A.9) yields the second inequality in (23). $\qquad$ Q.E.D.

<u>Claim 2.</u>   Given (A1) and (A2), the contracts $(\underline{\alpha},\underline{D})$ and $(\bar{\alpha},\bar{D})$ can be supported as a separating PBE if the non-negative profit constraints (15) and (16) hold with equality,

(24) $\quad \underline{\alpha} \geq \dfrac{\bar{\pi}' - \bar{\pi}^S}{\bar{\pi}' - \underline{\pi}^S}$,   and

(25) $\quad \bar{\alpha} \leq \dfrac{\underline{\pi}^S - \pi'}{\bar{\pi}^S - \underline{\pi}'}$.

<u>Proof of Claim 2.</u> Much of the proof follows the proof of Claim 1. The creditor again accepts the deviation (α',D') if (A.1) holds, and the $\bar{t}$-type and the $\underline{t}$-type do not have a strict incentive to deviate if

(A.10) $(1-\bar{\alpha})(\bar{\pi}^S-\bar{D}) \geq (1-\alpha')\left(\pi(\eta,\bar{t})-D'\right)$ and

(A.11) $(1-\underline{\alpha})(\underline{\pi}^S-\underline{D}) \geq (1-\alpha')\left(\pi(\eta,\underline{t})-D'\right)$,

respectively. Again, assign the belief

$$\mu(\alpha',D') = \begin{cases} 0 & \text{if} \quad \alpha' \geq 0 \\ 1 & \text{if} \quad \alpha' < 0 \end{cases}$$

to the creditor. The non-negative profit constraints become (A.4) and (A.5) when $\alpha' \geq 0$ and $\alpha' < 0$, respectively.

   Using the zero-profit condition (15), (A.10) holds if

(A.12) $\bar{\pi}^S - \pi(\eta,\bar{t}) \geq R - \left\{\alpha'\pi(\eta,\bar{t})+(1-\alpha')D'\right\}$,

and using (16), (A.11) holds if

(A.13) $\underline{\pi}^S - \pi(\eta,\underline{t}) \geq R - \left\{\alpha'\pi(\eta,\underline{t})+(1-\alpha')D'\right\}$.

Note that if $\alpha' \geq 0$, then (A.4) and assumption (A1) imply that the righthand sides of (A.12) and (A.13) are non-positive. Similarly, if $\alpha' < 0$, then (A.5) and (A1) imply that these quantities are non-positive. Thus, it suffices to show that

(A.14) $\bar{\pi}^S - \pi(\eta,\bar{t}) \geq 0$ and

(A.15) $\underline{\pi}^S - \pi(\eta,\underline{t}) \geq 0$.

If (A2) holds set $\eta = 1$; if (A2') holds set $\eta = 0$.

To complete the proof, substitute (15) and (16) with equality into (13) and (14). This yields (24) and (25).   Q.E.D.

Proposition 3. Given (A1), (A2), and (*), the contract (α, D) can be supported as a pooling FGP equilibrium if and only if α satisfies (12) and (α, D) satisfies the zero-profit condition (1).

Proof of Proposition 3. Given Claim 1, the proof of Proposition 3 requires that the argument in the text leading to Proposition 1 be strengthened to include: (i) deviations with a pooling consistent interpretation, (ii) deviations with a $\bar{t}$-separating consistent interpretation that earn strictly positive profit for the creditor, and (iii) deviations with a $\underline{t}$-separating consistent interpretation that earn strictly positive profit for the creditor. We treat these in turn. Not surprisingly, none of these has any cutting power.

(i) As argued in the text, any pooling FGP equilibrium must involve zero expected profit for the creditor. Similarly, the creditor accepts deviations only if they earn non-negative expected profit. This implies that there can be no deviation offered by both types: if one type has a strict incentive to deviate then the other has a strict incentive not to deviate, because aggregate profits are constant.

(ii) Section 5 considers deviations offered by the $\bar{t}$-type that earn zero profit for the creditor. Here we consider positive-profit deviations. Define

(A.16) $\bar{r} \equiv \alpha' \bar{\pi}^s + (1-\alpha)D' - R;$

$\bar{r} \geq 0$ by (2). Following the derivation of (5) and (6) yields

(A.17) $\alpha(1-\theta)(\bar{\pi}^P - \underline{\pi}^P) \geq \bar{\pi}^P - \bar{\pi}^S + \bar{r}$,   and

(A.18) $\alpha'[\bar{\pi}^S - \underline{\pi}'] \leq \alpha\theta(\bar{\pi}^P - \underline{\pi}^P) + [\underline{\pi}^P - \underline{\pi}' + \bar{r}]$.

Given α and $\bar{r}$, there exists an α' satisfying (A.18). Therefore, there exists a deviation if (A.17) holds. Not surprisingly, if (A.17) holds for $\bar{r} > 0$ then it holds for $\bar{r} = 0$. Thus, the necessary condition given in the text (that (5) fail), which rules out all zero-profit deviations of this form, also suffices to rule out all positive-profit deviations of this form.

(iii) The analogous argument shows that if there exists a positive-profit deviation offered by the $\underline{t}$-type then there exists a zero-profit deviation of the same form. Thus, the necessary condition given in the text (that (10) fail) again covers all deviations of this form.   Q.E.D.

<u>Proposition 4.</u>   Given (A1) and (A2), the contracts $(\underline{\alpha},\underline{D})$ and $(\bar{\alpha},\bar{D})$ specified in Claim 2 can be supported as FGP equilibria if and only if (*) fails or holds with equality.

<u>Proof of Proposition 4.</u> Given Claim 2, the proof of Proposition 4 requires that the argument in the text leading to Proposition 2 be strengthened to include: (i) a proof that the non-negative profit constraints (15) and (16) must hold with equality in a separating FGP equilibrium; (ii) deviations with either $\bar{t}$-separating or $\underline{t}$-separating

consistent interpretations; and (iii) deviations with a pooling consistent interpretation that earn strictly positive expected profit for the creditor.

(i) We show first that (15) and (16) must hold as equalities: the creditor must earn zero profit on each contract. Suppose, to the contrary, that (15) holds strictly. Then the deviation (α',D') has a $\bar{t}$-separating consistent interpretation, because (α',D') can be chosen to satisfy:

$$(A.19) \qquad \alpha'\bar{\pi}^S + (1-\alpha')D' = R \quad \text{and}$$

$$(A.20) \qquad (1-\underline{\alpha})(\underline{\pi}^S - \underline{D}) = (1-\alpha')[\underline{\pi}'-D'].$$

Substituting (A.19) into (A.20) yields

$$(A.21) \qquad \alpha'\left[\bar{\pi}^S - \underline{\pi}'\right] = (1-\underline{\alpha})(\underline{\pi}^S - \underline{D}) - \underline{\pi}' + R,$$

which determines α'; D' is then determined by (A.19). The $\bar{t}$-type clearly prefers to deviate because its financing cost is reduced to R while its product-market profit is unchanged. Limited liability requires that $(1-\alpha')(\bar{\pi}^S - D') \geq 0$, but this is implied by (A.19). Thus, in a separating FGP equilibrium, the creditor makes zero profit on the $\bar{t}$-type. An analogous argument establishes that (16) also holds with equality.

(ii) Deviations with $\bar{t}$-separating or $\underline{t}$-separating consistent interpretations share the property that the information conveyed to firm B by the deviation is the same as the information conveyed by the

putative equilibrium. Thus, product-market profits to the type in question do not change. Because (15) and (16) hold with equality, there is no scope for more attractive financing terms. These two observations together imply that deviations with such consistent interpretations do not exist.

(iii) It remains to show that there do not exist deviations with a pooling consistent interpretation that earn strictly positive expected profit for the creditor. The argument is simple: if (α', D') is such a positive-profit deviation, then there exists a zero-profit deviation with a pooling consistent interpretation, because the firm can reduce the financing cost $\alpha' \tilde{\pi}^p + (1-\alpha')D'$ to R by reducing $(1-\alpha')D'$ while holding α' constant, and the creditor still will accept. Q.E.D.

Footnotes

[1]We use the term "creditor" for any agent who finances the firm's project, regardless of whether the financing involves debt or equity.

[2]Recall that we are deliberately vague about the post-contract subgame that generates the payoff $\pi(q,t)$, and hence we do not specify particular strategy spaces for the players in this subgame.

[3]It is possible for a single deviation to have several different consistent interpretations. In multi-audience signaling games, this possibility implies that different uninformed players could believe different consistent interpretations. We avoid this issue by assuming that cheap talk is available to firm A at the contract-offer stage. This allows firm A to select a single consistent interpetation for the creditor and firm B through the use of a credible neologism; see Farrell [1986].

[4]Our no-bankruptcy assumption, (A0), guarantees that the limited liability constraints cannot bind in a pooling perfect Bayesian equilibrium: the creditor accepts the deviation $(0,R)$ for any belief μ, so the payoff to the deviating type, $\pi(\eta,s) - R > 0$, strictly exceeds the putative equilibrium payoff of zero.

[5]Madrigal and Tan [1986], for instance, show that *if* a competitive equilibrium outcome in an RSW game can be supported (by appropriate out-of-equilibrium beliefs) as a perfect Bayesian equilibrium in the analogous signaling game, then it can be supported as an FGP equilibrium in the signaling game. And Stiglitz and Weiss [1986] provide conditions that guarantee that an RSW equilibrium outcome can be supported as a PBE in the analogous signaling game.

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